Nanomechanical characterization of the Kondo charge dynamics in a carbon nanotube
Supplementary Material

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(Dated: February 1, 2018)

CONTENTS

I. Capacitive sensitivity
II. Gate voltage dependence of the resonance frequency
References

I. CAPACITIVE SENSITIVITY

The sensitivity of the gate capacitance $C_g$ on the nanotube position $z$ depends on several parameters that are experimentally not well-known. In the fit procedure for the results on device A discussed in the main manuscript and below, we have therefore used $dC_g/dz$ as a free parameter, resulting in

$$\frac{dC_g}{dz} = -27 \frac{z \text{F}}{\text{nm}}$$

In the following, we estimate this parameter independently and discuss error sources.

A typical simplified model for the gate capacitance of a suspended carbon nanotube is that of a metallic cylindrical beam above a conductive plane,

$$C_{th}^g = \frac{2 \pi \varepsilon_0 L}{\ln \left( \frac{2z}{r} \right)}$$

where $L$ is the beam length, $z$ the distance between beam and plane, and $r$ is the beam radius. We can estimate the length of our nanotube $L$ with the distance between the contact electrodes, $L = 700 \text{ nm}$. Since we do in general not image our devices before measurement to avoid contamination, and since this particular device did not survive a subsequent attempt at Raman spectroscopy, we do not know the precise nanotube position or orientation, and larger values are well possible.

The radius of the nanotube is similarly hard to quantify. Measurements of the single electron magnetic moment on the same device result in $R = 2.2 \text{ nm}$ [1], though the used model (as published originally in [2]) likely does not capture the entire required physics [1]. The distance between nanotube and gate can be approximated as the sum of gate oxide thickness and contact electrode thickness, $z = 340 \text{ nm}$. This does not take possible slack of the nanotube into account, nor the layer structure of 200 nm vacuum (electrode thickness plus etch depth) and 140 nm SiO$_2$ (remaining oxide) with $\varepsilon_r = 3.9$. With above approximations, we obtain

$$C_{th}^g = 6.8 \text{ aF}$$

compared to the value $C_g = 2.4 \text{ aF}$ from transport spectroscopy.

The localized electronic system of the quantum dot does not occupy the entire length of the nanotube; we use this to define an electronic length $L_{el} < L$ such that the calculated gate capacitance becomes equal the measured one,

$$L_{el} = \frac{C_g}{C_{th}^g} L = 250 \text{ nm}.$$
Using this length in Eq. S-2 and assuming a uniform deflection along the quantum dot, we obtain at \( z = 340 \text{ nm} \)

\[
\left. \frac{dC_g}{dz} \right|_{z=340 \text{ nm}} = -1.2 \frac{2F}{\text{nm}},
\]

approximately smaller by a factor 20 than the fit result. A likely conclusion is that our suspended nanotube lies closer to the gate than expected, however, given the many approximations and unknowns no definite statement can be made.

II. GATE VOLTAGE DEPENDENCE OF THE RESONANCE FREQUENCY

Both for displaying the observed resonance frequency behaviour more clearly and for improving the numerical stability of the fit results, as first step a linear contribution is subtracted from the raw resonance peak positions in Fig. 2(a) of the main text. The resulting resonance frequency shift, converted to angular frequency, is plotted in Figs. 2(d) and 3(a) of the main text (data points):

\[
\tilde{\omega}_0(V_g) = 2\pi f_0(V_g) - (a + bV_g) , \quad a = 2041 \times 10^6 / \text{s}, \quad b = 261.9 \times 10^6 / \text{Vs}
\]

As detailed in [3], the gate voltage dependence of the mechanical resonance frequency in Coulomb blockade consists of essentially three terms:

a), a continuous increase, corresponding to the continuous increase of the gate charge and the respective tension component,

b), a step function, corresponding to the stepwise increase of the quantum dot charge and the respective tension component, and

c), “frequency dips” corresponding to the softening of the spring constant by charge fluctuation whenever single electron tunneling is possible.

Approximating term a) as linear within the evaluated gate voltage region, we obtain as model

\[
\omega_0(V_g) = a' + b'V_g + \kappa \langle N \rangle(V_g) + \Delta \omega_0(V_g).
\]

As already discussed in the main text, we assume the density of states on the quantum dot to be a sequence of two equal-width Lorentzian peaks, aligned with the Fermi edge of the grounded drain contact at gate voltages \( V_{g1} \) and \( V_{g2} \), and separated by a corresponding charging energy. Tunnel barrier transparencies for both contacts are equal and energy-independent. In zero temperature approximation, the Fermi distribution in both leads becomes a step function, offset by the small source-drain voltage \( V_{sd} \). Following [4], as an example, the tunnel rate into (+) the level corresponding to Coulomb oscillation 1 from the contact L (source) then becomes

\[
\Gamma_{1L}^+(V_g) = a \left( 1 + \frac{1}{\pi} \arctan \left( \frac{2e(V_{sd} + (-V_g + V_{g1})\alpha)}{\Gamma \hbar} \right) \right)
\]

Writing \( \Gamma_{1/2}^+ = \Gamma_{1/2L}^+ + \Gamma_{1/2R}^+ \) and treating the occupation of the two levels as independent of each other, the time-averaged charge occupation of the quantum dot \( \langle N \rangle(V_g) \) becomes

\[
\langle N \rangle(V_g) = N_0 + \frac{\Gamma_1^+}{\Gamma_1^+ + \Gamma_1^-} + \frac{\Gamma_2^+}{\Gamma_2^+ + \Gamma_2^-}
\]

For the derivation of the frequency “dips”, term c), we then use the expression from [3, 4],

\[
\Delta \omega_0 = \frac{V_g(V_g - V_{CNT})}{2m\omega_0C_\Sigma} \left( \frac{dC_g}{dz} \right)^2 \left( 1 - \frac{e}{C_g} \frac{\partial \langle N \rangle}{\partial V_g} \right).
\]

\( C_g \) and \( C_\Sigma \) are obtained from Coulomb blockade (CB) measurements; \( m \) is estimated from radius and length of the nanotube (see above).
### Supplementary Table S-I

Parameters for Fig. 3 of the main text.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Source</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Bias voltage</td>
<td>$V_{bi}$</td>
<td>set in measurement</td>
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<tr>
<td>Nanotube length</td>
<td>$L$</td>
<td>device geometry</td>
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<tr>
<td>Nanotube radius</td>
<td>$R$</td>
<td>from [1]</td>
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<td>Nanotube mass</td>
<td>$m$</td>
<td>from $L$ and $R$</td>
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<tr>
<td>Gate capacitance</td>
<td>$C_g$</td>
<td>CB evaluation</td>
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<tr>
<td>Total capacitance</td>
<td>$C_\Sigma$</td>
<td>CB evaluation</td>
<td>44 aF</td>
</tr>
<tr>
<td>Gate lever arm</td>
<td>$\alpha$</td>
<td>CB evaluation</td>
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<td>Initial electron number</td>
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<td>CB evaluation</td>
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<td>Approx. resonance frequency</td>
<td>$\omega_0$</td>
<td>Fig. 2</td>
<td>$3.13 \times 10^9$/s</td>
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<td>Lifetime broadening of levels</td>
<td>$\Gamma$</td>
<td>fit result</td>
<td>$3.41 \times 10^{12}$/s</td>
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<tr>
<td>Constant frequency offset</td>
<td>$a + a'$</td>
<td>fit result</td>
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<tr>
<td>Linear frequency increase</td>
<td>$b + b'$</td>
<td>fit result</td>
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<tr>
<td>Dot charge-related frequency increase</td>
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<td>fit result</td>
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<td>Position CB oscillation 1</td>
<td>$V_{g1}$</td>
<td>fit result</td>
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<tr>
<td>Position CB oscillation 2</td>
<td>$V_{g2}$</td>
<td>fit result</td>
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<td>Capacitive sensitivity</td>
<td>$dC_g/dz$</td>
<td>fit result</td>
<td>$-27 \text{ aF/nm}$</td>
</tr>
</tbody>
</table>

In a first fitting step the positions of the Coulomb blockade oscillations, $V_{g1}$ and $V_{g2}$, are fixed, and the bare tunnel couplings, the level broadening $\Gamma$, the capacitive sensitivity $dC_g/dz$, $\kappa$, $a'$, and $b'$ are used as free parameters. In a subsequent second fitting step, the bare tunnel couplings and the level broadening $\Gamma$ are fixed and the capacitive sensitivity $dC_g/dz$, $\kappa$, $a'$, $b'$, $V_{g1}$, and $V_{g2}$ are used as free parameters.

The resulting values are summarized in Table S-I.

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